Write your name here


## Mathematics <br> Paper 3 (Calculator)

Higher Tier
4BNQMF"TTFTTNFOU.BUFSJBMTo*TTVF
Paper Reference
Time: 1 hour 30 minutes
1MA1/3H

You must have: Ruler graduated in centimetres and millimetress, Total Marks protractor, pair of compasses, pen, HB pencil, eraser, calculator.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
-     - there may be more space than you need.

Calculators may be used.


- If your calculator does not have a a button, take the value of $a$ to be 3.142 unless the question instructs otherwise.
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working out.


## Information

- The total mark for this paper is 80
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

Read each question carefully before you start to answer it.

- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.


## Answer ALL questions.

## Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The diagram shows a trapezium $A B C D$ and two identical semicircles.


The centre of each semicircle is on $D C$.
Work out the area of the shaded region.
Give your answer correct to 3 significant figures.
Area of trapezium $\rightarrow \frac{(a+b)}{2} \times h \rightarrow\left(\frac{12+28}{2}\right) \times 14=280 \mathrm{~cm}^{2}$ Area of Semi-circle $\rightarrow \frac{1}{2} \times \pi \times r^{2} \rightarrow 2 \times\left(1 / 2 \times \pi \times 3^{2}\right)=9 \pi_{\mathrm{cm}^{2}}$ Trapezium area - shaded area $=280-9 \pi=251.726 \mathrm{~cm}^{2}$

$$
=252 \mathrm{~cm}^{2} \quad(35 t)
$$

2 Asif is going on holiday to Turkey.
The exchange rate is $£ 1=3.5601$ lira.
Asif changes $£ 550$ to lira.
(a) Work out how many lira he should get.

Give your answer to the nearest lira.


Asif sees a pair of shoes in Turkey.
The shoes cost 210 lira.
Asif does not have a calculator.
He uses $£ 2=7$ lira to work out the approximate cost of the shoes in pounds.
(b) Use $£ 2=7$ lira to show that the approximate cost of the shoes is $£ 60$

$\underline{\underline{E 6 O}}=210 \mathrm{lira}$
(c) Is using $£ 2=7$ lira instead of using $£ 1=3.5601$ lira a sensible start to Asif’s method to work out the cost of the shoes in pounds?

You must give a reason for your answer.
Yes, it is a sasible start because he con now do the calculation withent a calculator. $t 2=7$ lira $50 \in 1=3.5$ lira Actual conversion is $\epsilon 1 \geq 3.5601$ lira. So good estimate (1)
(Total for Question 2 is $\mathbf{5}$ marks)

3 Here are the first six terms of a Fibonacci sequence.

$$
\begin{array}{llllll}
1 & 1 & 2 & 3 & 5 & 8
\end{array}
$$

The rule to continue a Fibonacci sequence is,
the next term in the sequence is the sum of the two previous terms.
(a) Find the 9 th term of this sequence.
$7^{\text {th }}$ form $\rightarrow 5+8=13$
8 th term $\rightarrow 8+13=21$
qm $\mathrm{tum} \rightarrow 13+21=34$

The first three terms of a different Fibonacci sequence are

$$
a \quad b \quad a+b
$$

(b) Show that the fth term of this sequence is $3 a+5 b$

4 th $\rightarrow(a+b)+b=a+2 b$
$5^{\text {ch }} \rightarrow(a+2 b)+(a+b)=2 a+3 b$
$6^{\text {th }} \rightarrow(2 a+3 b)+(a+2 b)=3 a+5 b$

Given that the 3 rd term is 7 and the 6th term is 29 ,
(c) find the value of $a$ and the value of $b$.
(1) $a+b=7$
(1) $\times 3 \rightarrow 3 a+3 b=21$
(2) $3 a+5 b=29$
(2) $\longrightarrow 3 a+5 b=27$

$$
0 a-2 b=-8
$$

$a+b=7$
$b=4$
$a+4=7$
$a=3$

4 In a survey, the outside temperature and the number of units of electricity used for heating were recorded for ten homes.

The scatter diagram shows this information.

Number of units used


Molly says,
"On average the number of units of electricity used for heating decreases by 4 units for each ${ }^{\circ} \mathrm{C}$ increase in outside temperature."
(a) Is Molly right?

Show how you get your answer.

$$
(2,80)(18,44) \rightarrow \frac{44-80}{18-2}=\frac{-36}{16}=-2.25 \text { units per }{ }^{\circ} \mathrm{C}
$$

She is incorrect because the number of uniat of electricity used decreases b) 2.5 units each time there is a $1^{\circ} \mathrm{C}$ increase. N of 4 units.
(b) You should not use a line of best fit to predict the number of units of electricity used for heating when the outside temperature is $30^{\circ} \mathrm{C}$.

Give one reason why.
Its extrapolation. The line of best fit dosent reach $30^{\circ} \mathrm{C}$. Since it is outside the range of data available it will be unreliable (1) to use the lime of best fit for this.
(Total for Question 4 is $\mathbf{4}$ marks)

5 Henry is thinking of having a water meter.
These are the two ways he can pay for the water he uses.
Water Meter
A charge of $£ 28.20$ per year
plus
91.22 p for every cubic metre of water used
1 cubic metre $=1000$ litres

## No Water Meter

A charge of $£ 107$ per year

Henry uses an average of 180 litres of water each day.
Use this information to determine whether or not Henry should have a water meter.
$180 \times 365$ days $=65,700$ litres used per year.
$\frac{65,700}{1000}=65.7$ cubic meters $\rightarrow 65.7 \mathrm{~m}^{3}$
with meter $\rightarrow t^{2} 8.20+(65.7 \times 60.9122)=t 88.13154$ without meter $\rightarrow$ - 107

So its better to have a meter as it works out cheaper.

6 Liz buys packets of coloured buttons.
There are 8 red buttons in each packet of red buttons.
There are 6 silver buttons in each packet of silver buttons.
There are 5 gold buttons in each packet of gold buttons.
Liz buys equal numbers of red buttons, silver buttons and gold buttons.
How many packets of each colour of buttons did Liz buy?
Work out LCM of 8,6 and 5 .


5
$2 \times 2 \times 2 \times 3 \times 5=120$
$\mathrm{red} \rightarrow \frac{120}{8}=15$ packets
Silver $\rightarrow \frac{120}{6}=20$ packets

$$
\text { gold } \rightarrow \frac{120}{5}=24 \text { packets }
$$

15 packets of red buttons 20 packets of silver buttons 24 packets of gold buttons
(Total for Question 6 is $\mathbf{3}$ marks)

7 The cumulative frequency table shows the marks some students got in a test.

| Mark $(\boldsymbol{m})$ | Cumulative frequency |
| :---: | :---: |
| $0<m<10$ | 8 |
| $0<m<20$ | 23 |
| $0<m<30$ | 48 |
| $0<m \leq 40$ | 65 |
| $0<m<50$ | 74 |
| $0<m \leq 60$ | 80 |

(a) On the grid, plot a cumulative frequency graph for this information.

(2)
(b) Find the median mark.

> Median Cumulative trequency $=40$
> Median $=27.5$
27.5

Students either pass the test or fail the test.
The pass mark is set so that 3 times as many students fail the test as pass the test.
(c) Find an estimate for the lowest possible pass mark.

$$
\begin{aligned}
& \text { Pass : tail }= 1: 3 \\
& 20: 60 \text { because total YO students }
\end{aligned}
$$

Only top 20 pass, 50 marks above cumulative frequency $=60$
Pass mark $=36.5$

8 Write 0.000068 in standard form.
$0 \sqrt{000068} \rightarrow 6.8 \times 10^{-5}$
$6.8 \times 10^{-5}$
(Total for Question 8 is 1 mark)

9 (a) Factorise $y^{2}+7 y+6$

$$
\begin{gathered}
y^{2}+7 y+6 \rightarrow(y+1)(y+6) \\
1 \times 6=6 \\
1+6=7
\end{gathered}
$$

(b) Solve $6 x+4>x+17$

$$
\begin{align*}
& 6 x+4>x+17 \\
& 5 x>13  \tag{2}\\
& x>\frac{13}{5} \\
&=
\end{align*}
$$

$$
x>13 / 5
$$

(c) $n$ is an integer with $-5<2 n \leqslant 6$

Write down all the values of $n$
$\div 2\binom{-5<2 n \leq 6}{-2.5<n \leq 3} \div 2$
So values of $n \rightarrow-2,-1,0,1,2,3$

$$
\begin{equation*}
-2,-1,0,1,2,3 \tag{2}
\end{equation*}
$$

(Total for Question 9 is 6 marks)

10 The function f is such that

$$
\mathrm{f}(x)=4 x-1
$$

(a) Find $\mathrm{f}^{-1}(x)$

$$
\begin{equation*}
y=4 x-1 \rightarrow y+1=4 x \rightarrow \frac{y+1}{4}=x \rightarrow \frac{f^{-1}(x)=\frac{x+1}{4}}{f^{-1}(x)=\frac{x+1}{4}} \tag{2}
\end{equation*}
$$

The function g is such that

$$
\mathrm{g}(x)=k x^{2} \text { where } k \text { is a constant. }
$$

Given that $\mathrm{fg}(2)=12$
(b) work out the value of $k$

$$
\begin{align*}
& g(2) \rightarrow k(2)^{2}=4 k \\
& f(4 k) \rightarrow 4(4 k)-1=12 \\
& 16 k-1=12  \tag{2}\\
& 16 k=13 \\
& k=13 / 16
\end{align*}
$$

$$
k=13 / 16
$$

11 Solve $x^{2}-5 x+3=0$
Give your solutions correct to 3 significant figures.
Quadratic formula $\rightarrow \frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

$$
a=1 \quad b=-5 \quad c=3
$$

$\frac{--5 \pm \sqrt{(-5)^{2}-4 \times 1 \times 3}}{2 \times 1} \rightarrow \frac{5 \pm \sqrt{13}}{2}$

$$
x=\frac{5+\sqrt{13}}{2}=\underline{4.30} \quad x=\frac{5-\sqrt{13}}{2}=0.697
$$

$$
x=0.697, x=4.30
$$

12 Sami asked 50 people which drinks they liked from tea, coffee and milk.
All 50 people like at least one of the drinks
19 people like all three drinks.
16 people like tea and coffee but do not like milk.
21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person likes only milk.
Sami selects at random one of the 50 people.
(a) Work out the probability that this person likes tea.



(b) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.
44 people like tea, out of which $5+16=21$ people only like one other dink. So 21/44
$13 A B C D$ is a rhombus.

$M$ and $N$ are points on $B D$ such that $D N=M B$.
$m$ Prove that triangle $D N C$ is congruent to triangle $B M C$.
$D N=M B$
$D C=B C$ because $A B C D$ is a rhombus so all sides have equal leyth. $\angle N D C=\angle M B C$ because they are base angles of triangle $D C D$, so equal.

The triangles NDC and BMC are ode up of two sides of same ley th and an equal angle. So they are congruent because thy follow the SAS rule.

14 (a) Show that the equation $x^{3}+4 x=1$ has a solution between $x=0$ and $x=1$

$$
\begin{aligned}
& x^{3}+4 x=1 \rightarrow x^{3}+4 x-1=0 \\
& \text { let } f(x)=x^{3}+4 x-1 \\
& x=0 \rightarrow+(0)=(0)^{3}+4(0)-1=-1 \\
& x=1 \rightarrow f(1)=(1)^{3}+4(1)-1=4
\end{aligned}
$$

Since there is a sign charge, there must be at least one root in the interval $0<x<1$ as the function is continous.
(b) Show that the equation $x^{3}+4 x=1$ can be arranged to give $x=\frac{1}{4}-\frac{x^{3}}{4}$

$$
x^{3}+4 x=1 \rightarrow 4 x=1-x^{3} \rightarrow x=\frac{1}{4}-\frac{x^{3}}{4}
$$

15 There are 17 men and 26 women in a choir.
The choir is going to sing at a concert.
One of the men and one of the women are going to be chosen to make a pair to sing the first song.
(a) Work out the number of different pairs that can be chosen.

## 17 men and 26 women

Total pairs $\rightarrow 17 \times 26=442$

Two of the men are to be chosen to make a pair to sing the second song.
Ben thinks the number of different pairs that can be chosen is 136
Mark thinks the number of different pairs that can be chosen is 272
(b) Who is correct, Ben or Mark?

Give a reason for your answer.

Ben is correct, because correct calculation is $(16 \times 17) \div 2=136$ Divide bs 2 because the men are chosen to make a pair so dosent matter which order they are picked in .e.g 'man $A$ and man $B$ ' is same as (1) 'man $B$ and man $A$ '. (Total for Question 15 is $\mathbf{3}$ marks)
$16 V A B C D$ is a solid pyramid.

$A B C D$ is a square of side 20 cm .
The angle between any sloping edge and the plane $A B C D$ is $55^{\circ}$
Calculate the surface area of the pyramid.
Give your answer correct to 2 significant figures.
Area of square base $\rightarrow 20 \times 20=400 \mathrm{~cm}^{2}$
Area of 1 triangle $\rightarrow 1 / 2 \times b \times h$

$(A C)^{2}=(20)^{2}+(20)^{2}$
$(A C)^{2}=800$
$A C=\sqrt{800}$
$A X=1 / 2 \quad \sqrt{800}=10 \sqrt{2}$
$x$ and $M$ ore midpoints


$$
\begin{aligned}
& \operatorname{sen} 55=\frac{v x}{10 \sqrt{2}} \\
& v x=10 \sqrt{2} \times \tan 55 \\
& v x=20.197 \mathrm{~cm}
\end{aligned}
$$



$$
(V M)^{2}=(20.197)^{2}+10^{2}
$$

area of one of the triangles $=\frac{1}{2} \times 20 \times 22.537$

$$
(V M)^{2}=507.921
$$

$$
=225.371 \mathrm{~cm}^{2}
$$

$$
V M=22.537 \mathrm{~cm}
$$

= height of

$$
\begin{aligned}
\text { Total area } & =\text { square base }+4 \text { triangle } \\
& =400+4(225.371) \\
& =1300 \mathrm{~cm}^{2}
\end{aligned}
$$

triangles

1300

17 Louis and Robert are investigating the growth in the population of a type of bacteria. They have two flasks A and B.

At the start of day 1 , there are 1000 bacteria in flask A.
The population of bacteria grows exponentially at the rate of $50 \%$ per day.
(a) Show that the population of bacteria in flask A at the start of each day forms a geometric progression.

Day $1=1000 \underset{\times 1.5}{\longrightarrow} \operatorname{Day} 2=1500 \underset{\times 1.5}{\longrightarrow} \operatorname{Day} 3=2250$
Geometric protrusion as common ratio is 1.5.

The population of bacteria in flask A at the start of the 10th day is $k$ times the population of bacteria in flask A at the start of the 6th day.
(b) Find the value of $k$.
$6^{\text {th }}$ day $7^{\text {th }} 8^{\text {th }} 9^{\text {th }} 10^{\text {th }}$

$n x(1.5)^{4}=k n \rightarrow 1.5^{4}=k \rightarrow k=5.0625$

At the start of day 1 there are 1000 bacteria in flask B.
The population of bacteria in flask B grows exponentially at the rate of $30 \%$ per day.
(c) Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.

18

$O M A, O N B$ and $A B C$ are straight lines.
$M$ is the midpoint of $O A$.
$B$ is the midpoint of $A C$.
$\overrightarrow{O A}=6 \mathbf{a} \quad \overrightarrow{O B}=6 \mathbf{b} \quad \overrightarrow{O N}=k \mathbf{b}$ where $k$ is a scalar quantity.
Given that $M N C$ is a straight line, find the value of $k$.

$$
\begin{aligned}
\overrightarrow{O M} & =\overrightarrow{M A}=1 / 2 \overrightarrow{O A}=3 a \\
\overrightarrow{A B} & =\overrightarrow{A O}+\overrightarrow{O B}=-6 a+6 b \\
\overrightarrow{M C} & =\overrightarrow{M A}+2(\overrightarrow{A B}) \\
& =3 a+2(6 b-6 a)=12 b-9 a \\
\overrightarrow{M N} & =\overrightarrow{M O}+\overrightarrow{O N}=-3 a+k b
\end{aligned}
$$

$$
\text { as MNC is a straight line, } \overrightarrow{M c} \text { is a mutiple of } \overrightarrow{M N}
$$

$$
\begin{aligned}
\overrightarrow{M C}=x \overrightarrow{M N} \rightarrow 12 b-9 a=x \times(4 b-3 a) \\
12 b-9 a=3(4 b-3 a)
\end{aligned}
$$

$$
x=3 \quad k=4
$$

